

Review of Trigonometric, Logarithmic, and Exponential Functions

In this tutorial, we review trigonometric, logarithmic, and exponential functions with a focus on those properties which will be useful in future math and science applications.

Trigonometric Functions

Geometrically, there are two ways to describe trigonometric functions:

Polar Angle

Envision the Unit Circle – now draw your version:



$$x = \cos \theta$$

$$y = \sin \theta$$

measure θ in radians:

$$\theta = \text{arc length} / \text{radius}$$

For example, $180^\circ = \pi r / r = \pi$ radians

$$\text{Radians} = [\text{degrees}/180] \pi$$

Right Triangle

Draw a right triangle with base angle θ , opposite side y , adjacent side x , and hypotenuse r

$$\sin \theta = \text{opposite/hypotenuse} = y/r$$

$$\cos \theta = \text{adjacent/hypotenuse} = x/r$$

$$\tan \theta = \text{opposite/adjacent} = y/x$$

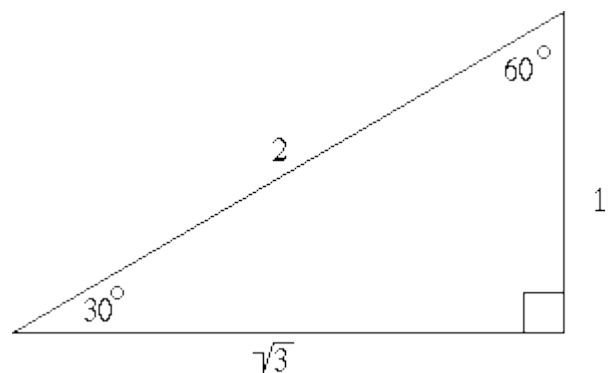
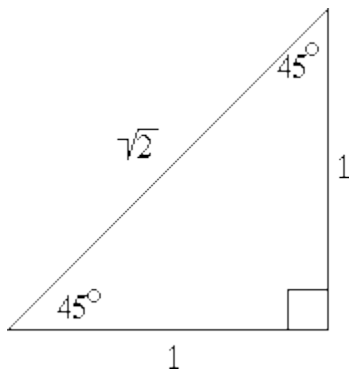
$$\csc \theta = 1/\sin \theta = r/y$$

$$\sec \theta = 1/\cos \theta = r/x$$

$$\cot \theta = 1/\tan \theta = x/y$$

Evaluating Trigonometric Functions

	0 rad	$\pi/6$ rad	$\pi/4$ rad	$\pi/3$ rad	$\pi/2$ rad
	0°	30°	45°	60°	90°
$\sin \theta$	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0
$\tan \theta$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	undefined



$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\cos(\theta + \pi) = -\cos \theta$$

$$\sin(\theta + \pi) = -\sin \theta$$

$$\sin(\theta + \pi/2) = \cos \theta$$

$$\cos(\theta + \pi/2) = -\sin \theta$$

$$\cos(\theta + 2\pi) = \cos \theta$$

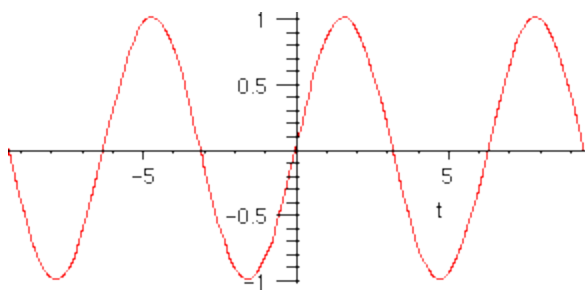
$$\sin(\theta + 2\pi) = \sin \theta$$

Trigonometric Identities

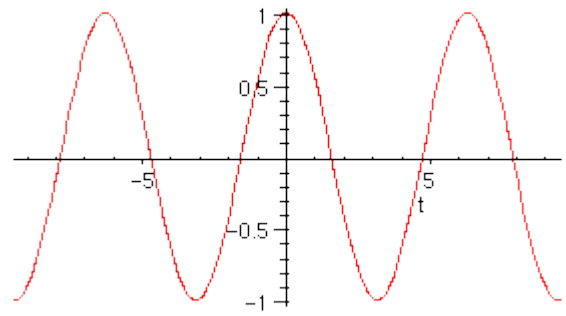
We list here some of the most commonly used identities:

1. $\cos^2 \theta + \sin^2 \theta = 1$
2. $\cos^2 \theta = \frac{1}{2}[1 + \cos(2\theta)]$
3. $\sin^2 \theta = \frac{1}{2}[1 - \cos(2\theta)]$
4. $\sin(2\theta) = 2\sin \theta \cos \theta$
5. $\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$
6. $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$
7. $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

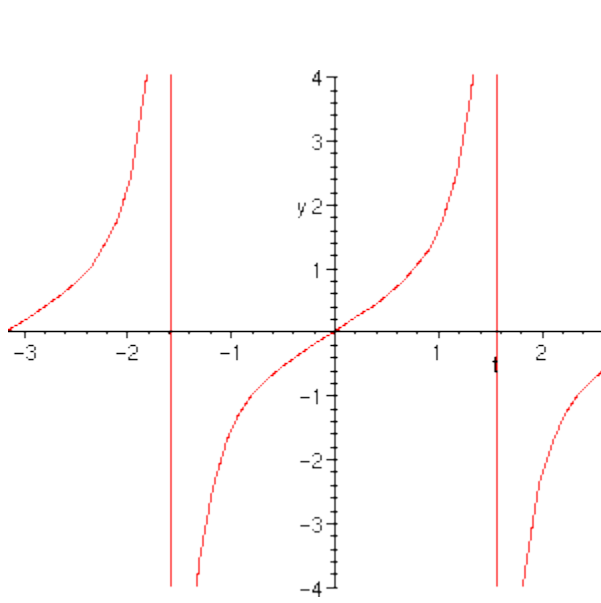
Graphs of Trigonometric Functions



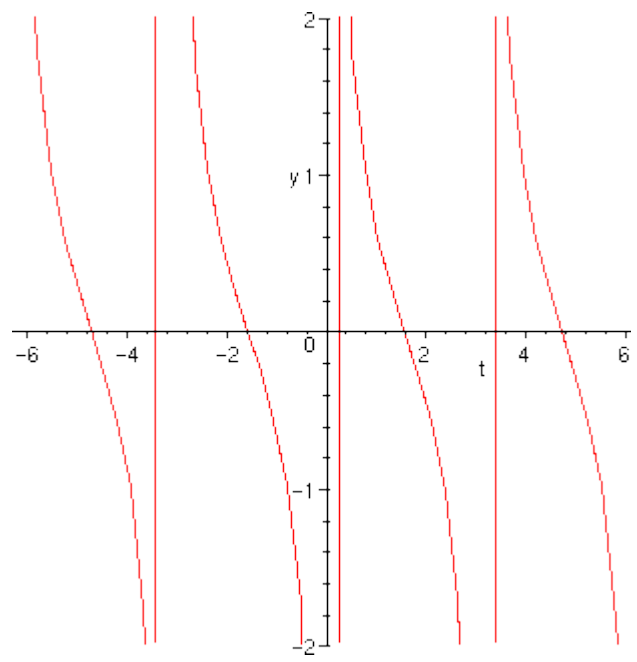
$\sin x$



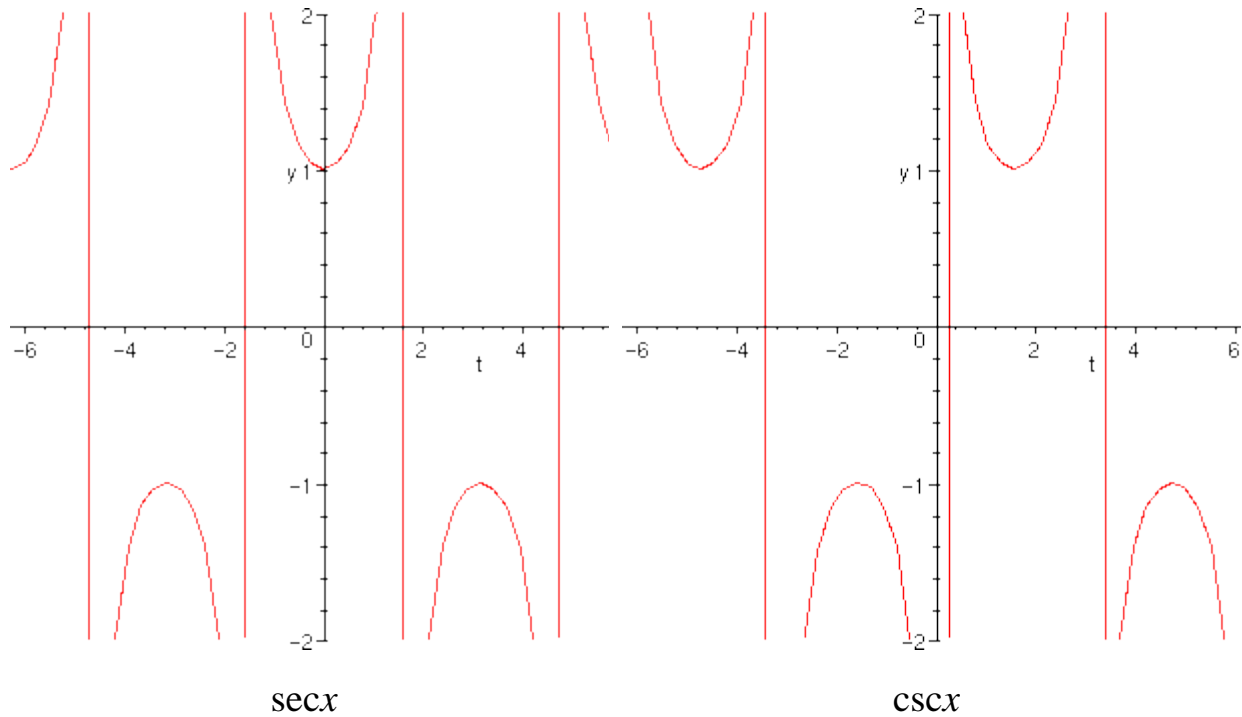
$\cos x$



$\tan x$



$\cot x$



Logarithmic and Exponential Functions

Logarithmic and exponential functions are inverses of each other:

$$Y = \log_b x \quad \text{if and only if} \quad x = b^y$$

$$Y = \ln x \quad \text{if and only if} \quad x = e^y$$

In words, $\log_b x$ is the exponent you put on base b to get x . Thus,

$$\text{Log}_b b^x = x \quad \text{and} \quad b^{\log_b x} = x$$

More Properties of Logarithmic and Exponential Functions

Notice the relationship between each pair of identities:

$$\text{Log}_b 1 = 0 \quad \text{or} \quad b^0 = 1$$

$$\text{Log}_b b = 1 \quad \text{or} \quad b^1 = b$$

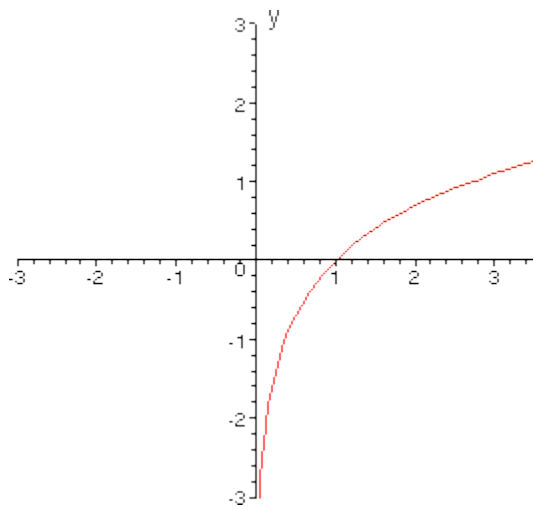
$$\log_b(1/c) = -\log_b c \quad \text{or} \quad b^{-m} = 1/b^m$$

$$\log_b ac = \log_b a + \log_b c \quad \text{or} \quad b^m b^n = b^{m+n}$$

$$\log_b(a/c) = \log_b a - \log_b c \quad \text{or} \quad b^m/b^n = b^{m-n}$$

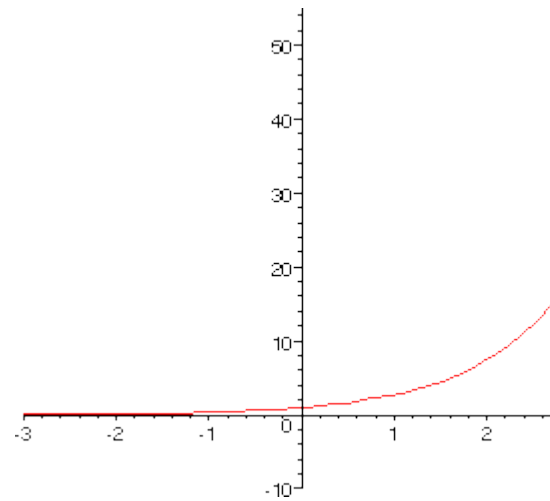
$$\log_b a^r = r \log_b a \quad \text{or} \quad (b^m)^n = b^{mn}$$

Graphs of Logarithmic and Exponential Functions



$f(x) = \ln x$

Notice that each curve is the reflection of the other about the line $y = x$.



$f(x) = e^x$

Limits of Logarithmic and Exponential Functions

1. $\lim_{x \rightarrow \infty} (\ln(x)/x) = 0$ ($\ln x$ grows more slowly than x)
2. $\lim_{x \rightarrow \infty} (e^x/x^n) = \infty$ for all positive integers n (e^x grows faster than x^n)
3. for $|x| \ll 1$, $\lim_{n \rightarrow \infty} (1 + (x/n))^n = e^x$.

NOW FOR THE PROBLEMS THAT WILL HELP PREPARE YOU FOR AP PHYSICS C

Sketch a graph of the function and fill in the blanks. Include two full periods.

1.

$$f(x) = 4\cos(x/3)$$

Centerline _____

Amplitude _____

Period _____

Increment _____

2.

$$f(x) = -2\tan(x/3)$$

Period _____

Vertical Asymptotes _____

3

$$f(x) = (1/2)\csc(x/3)$$

4.)

$$F(x) = (1/2)\cot(x/2 + \pi/6)$$

Period _____

Vertical Asymptotes _____

5.

$$Y = 4\sin(x/2 - \pi/2)$$

Centerline _____

Amplitude _____

Phase Shift _____

Period _____

Increment _____

6.

$$y = (1/2) \sec (2x + \pi/4)$$

7. $y = -3\sec x$

8.

$$y = 1 + 4\sin(x/3)$$

Centerline _____

Amplitude _____

Phase Shift _____

Period _____

Increment _____

9.

$$y = 1 - 2\csc 2x$$

10.

$$y = 4\cos(4x + \pi/2)$$

Centerline _____

Amplitude _____

Phase Shift _____

Period _____

Increment _____

11.

Chemco Manufacturing estimates that its profit P in hundreds of dollars is

$P = -2x^2 + 16x + 2$, where x is the number of units produced in thousands. How many units must be produced to obtain a maximum profit?

A. 4 units B. 32 units C. 3200 units D. 4000 units.

12.

Find the height of a tree on a hillside of slope 32° (from the horizontal). At a point 75 feet down the hill from the tree, the angle of elevation to the top of the tree is 48° .

13.

To approximate the length of a marsh, a surveyor walks 450 meters from point A to point B, turns 65° and walks 325 meters to point C. What is the length of the

marsh(AC)?

14.

Solve the system of equations:

$$7x + 12y = 63$$

$$2x + 3y = 15$$

15.

A contractor is hiring two trucking companies to haul 1600 tons of crushed stone for a highway construction project. The contract states that company A must haul 4 times as much

stone as the company B. Find the amount of stone hauled by each company.