Pre-Calculus **Name: ______________________________**

9.5 - Parametric Equations

Parametric curves are relations $(x(t), y(t))$ for which both x and y are defined as functions of a **third** variable, t. As in $x = f(t)$ and $y = g(t)$. Essentially, we now will have the ability to not only tell where an object is given a point (x, y), but also when the object is at that point (x, y).

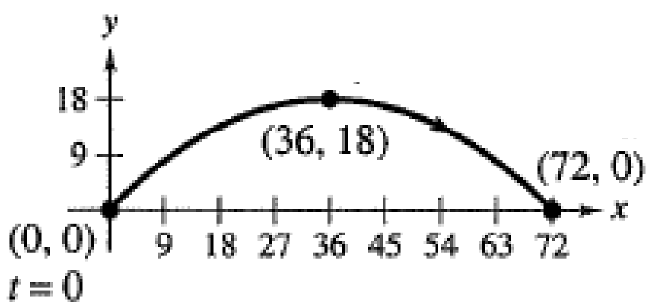
Given:

$$y = -\frac{x^2}{72} + x$$

With the Parameter:

$$x = 24\sqrt{2}t$$

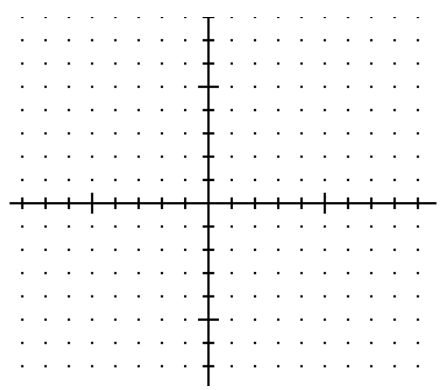
$$y = -16t^2 + 24\sqrt{2}t.$$



Sketch by hand. Indicate direction. Find the domain and range....

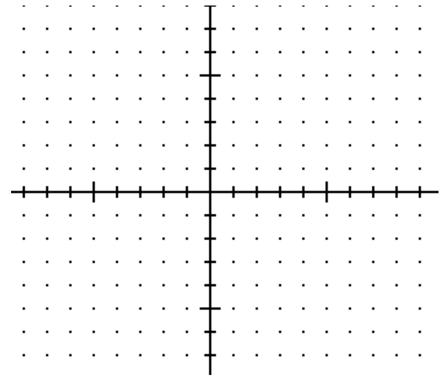
Example 1: Sketch the curve given by the parametric equations: $\begin{cases} x = t^2 - 4 \\ y = \dfrac{t}{2} \end{cases}$, for $-2 \le t \le 3$

t						
x						
y						



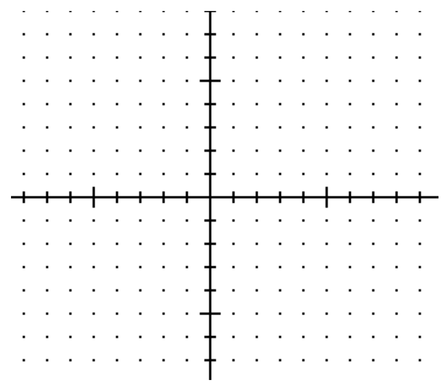
Example 2: A particle moving in the coordinate plane in such a way that $x(t) = 3t - 6$ and $y(t) = t - 4$ for $0 \le t \le 5$.

t						
x						
y						



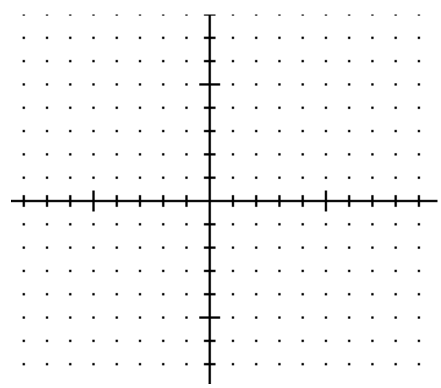
Example 3: Sketch the curve and the orientation given by $x(t) = 2 - t$ and $y(t) = t^2$ for $-2 \le t \le 3$

t						
x						
y						



Example 4: Sketch the curve and the orientation given by $x(t) = 2\cos\theta$ and $y(t) = 6\sin\theta$ for $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, 2\pi$.

T								
x								
y								



Use a graphing calucator....

Example 5 : Use a graphing calucator to graph the curves represented by the parametric equations. Tell which curves are functions.

a. $\begin{cases} x = t^2 \\ y = t^3 \end{cases}$

b. $\begin{cases} x = t \\ y = t^3 \end{cases}$

c. $\begin{cases} x = t^2 \\ y = t \end{cases}$

d. $\begin{cases} x = t^2 \\ y = 2 + t \end{cases}$, from $-3 \le t \le 3$

e. $\begin{cases} x = 4\sin\theta \\ y = 3\cos\theta \end{cases}$

Eliminate the parameter….

Example 6: Find the Cartesian (rectangular) equations of the curve. Identify the curve represented by the equations.

a. $\begin{cases} x = \dfrac{1}{\sqrt{t+1}} \\ y = \dfrac{t}{t+1} \end{cases}$

b. $\begin{cases} x = 3\cos\theta \\ y = 4\sin\theta \end{cases}$, for $0 \le \theta \le 2\pi$

c. $x = t - 2$ and $y = \dfrac{1}{t-1}$

d. $x = t^2 - 1$ and $y = t + 2$

e. $x = t^2 + 2$ and $y = t^2 - 1$

f. $x = 3 + 2\cos\theta$ and $y = 1 + \sin\theta$

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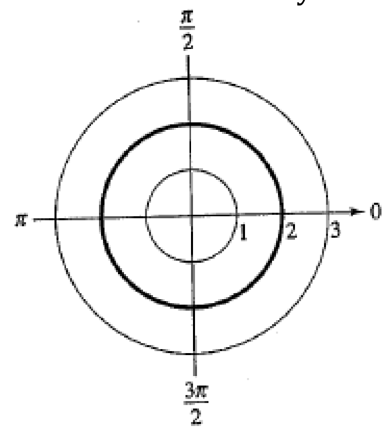
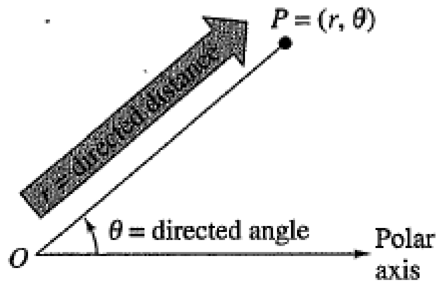
9.6 - Polar Coordinates

I. Polar coordinate system

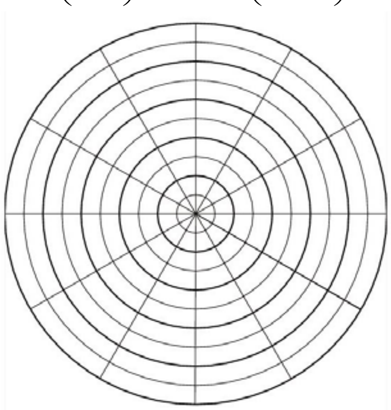


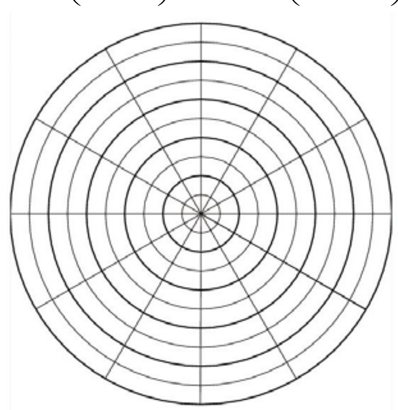


Plotting Points in the Polar Coordinate System

Example 1: Plot A and B on the first graph, then C and D on the second graph.

$A = \left(1, \frac{\pi}{4}\right)$ $B = \left(5, \frac{7\pi}{6}\right)$ $C = \left(3, -\frac{\pi}{2}\right)$ $D = \left(-4, \frac{2\pi}{3}\right)$

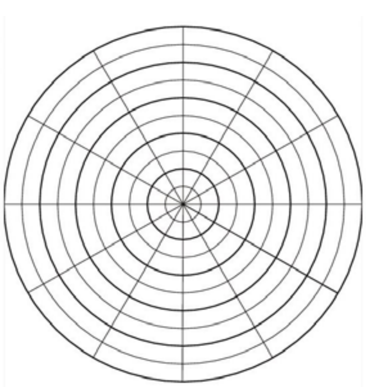




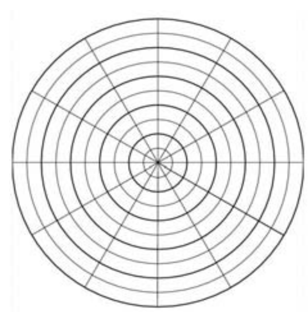
Unlike the Cartesian coordinate system, in which each point on the plane is expressed by a unique coordinate pair, each point in the coordinate plane can be represented by an infinite number of polar coordinate pairs.

Example 2.

A. Find and graph two polar coordinate pairs that represents the same point on the plane as $\left(2, \frac{\pi}{6}\right)$.



B. Find and graph polar coordinate pairs that represent the same point on the plane as $\left(-3, -\frac{2\pi}{3}\right)$ on the interval $[0, 2\pi]$.

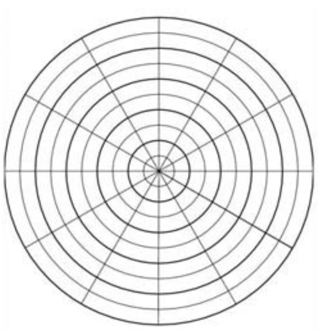


Coordinate Conversion

Recall: $x = r\cos\theta$ $y = r\sin\theta$ $r = \sqrt{x^2 + y^2}$ $\tan\theta = \dfrac{y}{x}$

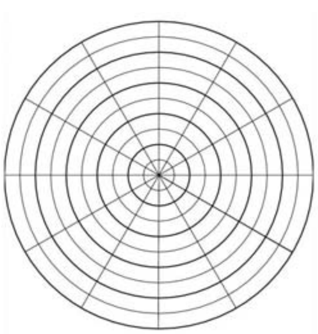
Example 3: Convert each point to rectangular coordinates. Graph the polar coordinates.

a. $(2, \pi)$ b. $\left(\sqrt{3}, \dfrac{\pi}{6}\right)$ c. $(-3, \pi)$ d. $\left(7, \dfrac{4\pi}{3}\right)$ e. $\left(-2, \dfrac{7\pi}{4}\right)$



Example 4: Rectangular-to-Polar Conversion. Graph the polar coordinates.

a. $(-1, 1)$ b. $(0, 2)$ c. (4, -4) d. $(-1, \sqrt{3})$

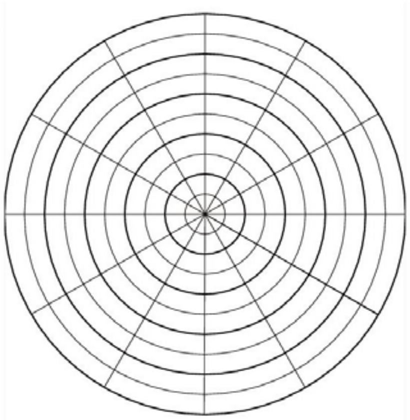


Equation Conversion

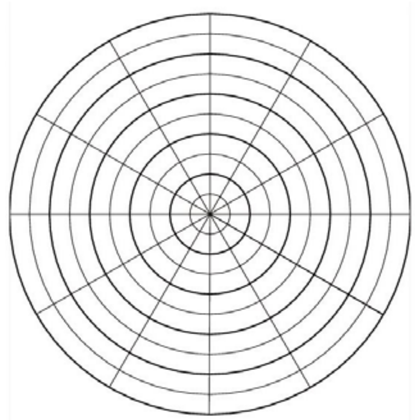
Example 5:
Describe the graph of each polar equation and find the corresponding rectangular equation.

a. r = 2 b. $\sin\theta = \dfrac{\pi}{3}$ c. $r = \sec\theta$

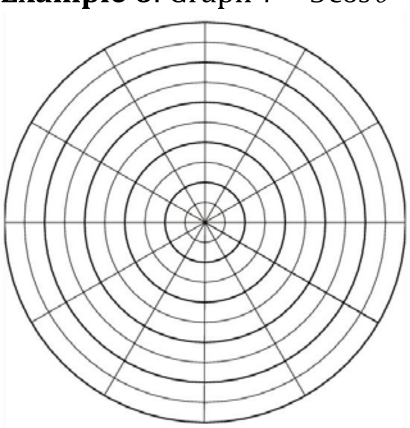
Example 6: $r = a\cos\theta$



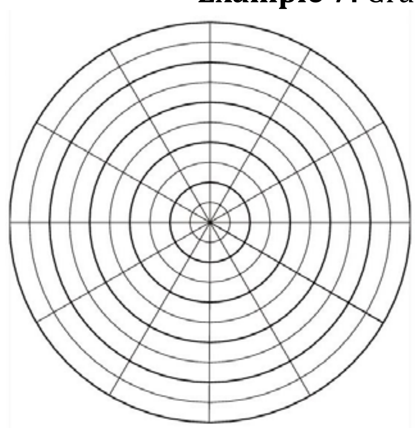
Example 7: $r = a\sin\theta$



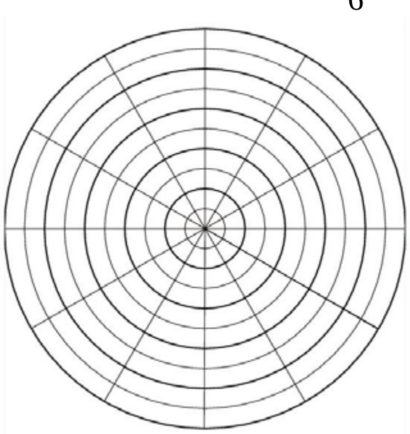
Example 6: Graph $r = 3\cos\theta$



Example 7: Graph $r = 5$



Example 6: Graph $\theta = -\frac{\pi}{6}$



Example 7: Graph $r = 4\cos 2\theta$